UNSTRUCTURED GRID CONVERGENCE STUDY

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SUMMARY

This paper focuses on convergence studies of CFD predictions using unstructured grids with our in-house finite volume unstructured RANSE solver ISIS-CFD. Computations have been performed with different grid sets generated with different approaches including similar grid generation with an unstructured grid generator and adaptive grid refinement. Numerical uncertainty is evaluated with Richardson extrapolation. Results obtained both with wall function and wall resolved approach are compared. The paper demonstrates that adaptive grid refinement is the most reliable approach for obtaining a grid independent solution.

1 INTRODUCTION

As CFD simulation is playing an important role in design procedures, industrial users are paying more and more attention to the reliability of numerical predictions. The most common practice for industrial users is to adjust numerical the setup (mesh density, physical model, numerical scheme, ...) and compare the numerical prediction with the measurement data. If the agreement is good enough, then, such setup will be used as guideline for applications under similar conditions. Although such procedure can help a user to obtain a good prediction, it cannot rigorously quantify the numerical uncertainty in a simulation. The recommended procedure to quantify numerical uncertainty is to perform a verification and validation exercise. There are three kinds of verification and validation exercises, namely code verification, solution verification and solution validation. Code verification aims at demonstrating that the governing equations are solved correctly. The most appropriate approach for this exercise is to modify the code by adding source terms such that numerical solution can be compared with a manufactured solution. With the manufactured solution, one can easily verify the order of convergence of the flow solver as the mesh is refined. If the observed order of convergence agrees with the expected theoretical order of convergence as the mesh is refined, the numerical implementation is considered as successful. Such code verification exercises are performed only once during the code development and do not need to be repeated for practical application. Unlike code verification, solution validation needs to be performed for every time if rigorous uncertainty quantification is required. It aims at quantifying errors due to the numerical approach. Different sources of error exist in numerical simulation. The most commons ones are space and time discretization error, convergence error, errors due to the boundary condition and to the computational domain, etc. We focus on errors due to the spatial discretization only in this paper. The most common and the most reliable approach to obtain spatial discretization error is to perform simulations on a set of similar grids, then apply an uncertainty estimation based on Richardson extrapolation. The success of such an uncertainty estimation procedure depends on how such similar grid sets are generated. Generating a similar grid set with structured grids is a trivial task. Unfortunately, industrial applications are commonly performed with unstructured grids. Ensuring grid similarity when generating a set of unstructured grid is nearly impossible. The situation is worse when wall function approach is used. Wackers and al. [3] have proposed an alternative to generate a set of similar grids for uncertainty estimation with adaptive grid refinement, where the mesh fineness is controlled by the grid refinement threshold. This approach has been successfully applied to an uncertainty estimation exercise for a simple 2D airfoil configuration as well as a more complex 3D ship flow test case both for global quantities (resistance) and for local quantities (velocity). The objective of this paper is to compare the approach proposed by Wackers et al. with a more classical grid generation approach using a grid generator for the uncertainty estimation exercise. Computations will be performed both with wall functions and with the wall-resolved approach with a very simple 2D configuration for which structured grid sets can also be generated for comparison.

2 NUMERICAL APPROACH

Computations have been performed with the ISIS-CFD flow solver developed by our team. Turbulent flow is simulated by solving the incompressible Reynolds-averaged Navier-Stokes equations (RANS). The flow solver is based on the finite volume method to build the
spatial discretization of the transport equations. The velocity field is obtained from the momentum conservation equations and the pressure field is extracted from the mass conservation constraint, or continuity equation, transformed into a pressure-equation. In the case of turbulent flows, additional transport equations for the modeled variables are discretized and solved using the same principles. The gradients are computed with an approach based on Gauss’s theorem. Non-orthogonal corrections are applied to ensure a formal first order accuracy. Second order accurate results are obtained on a nearly symmetric stencil. Inviscid fluxes are computed with a piecewise linear reconstruction associated with an upwinding stabilizing procedure which ensures a second order formal accuracy when no flux limiter is applied. Viscous fluxes are computed with a central difference scheme which guarantees a first order formal accuracy. We have to rely on mesh quality to obtain a second order discretization for the viscous term.

Free-surface flow is simulated with a multiphase flow approach. Incompressible and non-miscible flow phases are modeled through the use of conservation equations for each volume fraction of phase/fluid. An implicit scheme is applied for the time discretization. A second order Backward Differentiation Formula (BDF2) is employed for time-accurate unsteady computations. Velocity-pressure coupling is handled with a SIMPLE like approach. Free ship motion can be simulated with a 6 DOF module. Some degrees of freedom can be fixed as well. An analytical weighted mesh deformation approach is employed when free-body motion is simulated. An overset approach has been implemented recently. Several turbulence models ranging from one-equation models to the Reynolds stress transport models are implemented in ISIS-CFD. Most of the classical linear eddy-viscosity based closures like the Spalart-Allmaras one-equation model and the two-equation k-\(\omega\) SST model by Menter \cite{2}, for instance are implemented. A more sophisticated turbulence closure is also implemented in the ISIS-CFD solver, an explicit algebraic stress model (EASM) \cite{1}. The SST model is employed in the present study. A wall function boundary condition is implemented for the two-equation turbulence models.

Adaptive grid refinement is implemented in the ISIS-CFD flow solver with different refinement criteria. In the present study, we aim at resistance prediction only. To this end, the best refinement criterion is the pressure Hessian which is a tensor based on the second derivatives of the pressure field. In the refinement procedure, the mesh is refined until the module of a vector resulting from the tensor product between the cell size vector and the pressure Hessian tensor is smaller than a threshold value. Details can be found in \cite{4}.

3 SIMILAR MESH SET GENERATION

The success of an uncertainty estimation exercise depends strongly on how the mesh set employed for the computation is generated. Richardson extrapolation can give the expected result only when grid similarity of the mesh set is ensured. Two grids are considered as similar when the mesh orientation at the same position is the same, and the mesh size ratio between the two grids at any location is a constant. When using a wall function, grid similarity can no longer be ensured in the near-wall region, since the cell size in the wall normal direction should be kept unchanged. We employ the hexahedral unstructured grid generator Hexpress for mesh generation. With Hexpress, it is not possible to ensure grid similarity everywhere. Figure 1 shows two meshes with a refinement ratio of 2 generated with Hexpress. Although the mesh size ratio is about 2 everywhere, the location of the grid refinement interface is not exactly the same. The most annoying drawback of the mesh generator Hexpress is that the boundary layer thickness becomes smaller in the fine grid. Hence, in a convergence study using Hexpress, adding more fine grid cells usually does not help to obtain a better convergence behavior.

![Figure 1. Unstructured grid generated with Hexpress](image)

An alternative for generating a similar grid set is to perform adaptive grid refinement using different refinement thresholds. Figure 2 displays two meshes coming from grid adaptation. The pressure Hessian is employed as refinement criterion in this computation. The upper part is the coarse mesh, while the lower part is the fine mesh. The ratio for the refinement threshold is equal to 2. It can be seen that mesh size ratio is equal to 2 almost everywhere except in the boundary layer where we intentionally refine the mesh in the wall tangential direction only in order to preserve mesh quality in the viscous layers. The refined mesh is projected automatically on the surface of the real geometry. As the initial mesh is...
relatively coarse, the thickness of viscous layer in the
refined grid is much bigger compared to the mesh
generated directly with the mesh generator Hexpress
shown in figure 1. Moreover, it remains constant whatever
the degree of grid refinement. Hence, a turbulent boundary
layer can be better captured when using adaptive grid
refinement. The distance to the wall for the first grid cell is
specified with the same value corresponding to about
y+=30 when generating the grids. The different cell sizes
observed in figures 1 and 2 are the result of mesh
optimization performed by the mesh generator Hexpress.
The meshes shown in figures 1 and 2 are designed for wall
function computation. When generating grid sets for wall-
resolved computations with the mesh generator, it is
possible to adjust the distance to the wall for the first grid
cell such that grid similarity is also ensured in the near-
wall region. However, as refinement will not be applied in
the wall normal direction, grid similarity near the wall is
impossible to ensure with adaptive grid refinement. Hence,
we decide to also generate a grid set with the same distance
to the wall, corresponding to about y+=0.2 when using the
grid generator approach.

![Figure 2. Mesh obtained with adaptive grid refinement](image)

### 4 CONVERGENCE STUDY RESULTS

The test case investigated in the present paper is a simple
ellipse geometry with an aspect ratio of 1:5 in two
dimensions. The computational domain is defined by
-6L<x<18L, -4L<y<4L. L being the chord of the ellipse.
The center of the ellipse is located at (0,0). The pressure is
set to 0 at the outlet boundary x=18L, while a far field
boundary condition with constant velocity is imposed at
the remaining outer boundaries. The Reynolds number
based on the chord length is 1.0e6.

#### 4.1 WALL FUNCTION SIMULATIONS

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<tr>
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<th>100Fxp</th>
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</table>

Table 1: Wall modeled prediction with adaptive refinement

Table 1 presents the results obtained with adaptive grid
refinement. N is the number of grid cells. Fxp is the
pressure resistance, while Fx is the total resistance. Both
are normalized values using the chord length and far field
velocity. “p” is the observed order of convergence. U_e
is the extrapolated result with the observed order of
convergence, while U_e2 is the extrapolated value obtained
with assumed second order accuracy of the finite volume
flow solver. As the mesh in the wall normal direction is not
refined, it is expected that monotonic convergence behavior can not be obtained for the friction resistance. For
this reason, it is not shown in the table. For the computation with adaptive refinement, the threshold value is
used directly as an indication of the mesh size. The observed order of convergence is not too far from the
expected second order accuracy both for the pressure resistance and for the total resistance. This indicates that
the grid convergence study is successful. The prediction obtained with the finest grid differs from the extrapolated solution U_e2 by about 0.5% for the pressure resistance and
0.2% for the total resistance. Based on the extrapolated value U_e2 for pressure resistance and total resistance, the
expected friction resistance is 0.005051.

<table>
<thead>
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Table 2: Wall function prediction with mesh generation

Table 2 shows the predicted pressure resistance Fxp, friction resistance Fxv and the total resistance Fx. N1 is the
number of mesh points in the X-direction of the initial
mesh generated by Hexpress. 1/N1 can be used as the mesh
size for the Richardson extrapolation. With N1=72, the
coarsest grid contains 3 cells in the initial mesh. The initial
mesh is refined by 5 levels in Hexpress, resulting in 96
cells per chord length. One more level of refinement is applied near the leading edge and the trailing edge. The mesh size is about L/192 in those regions. The first grid is too coarse. Starting from grid 2 until grid 8, the predicted pressure resistance and the total resistance increase monotonously and approach the expected solution given in table 1. However, even on grid 8, the pressure resistance is still 9.5% smaller than the extrapolated solution \( U_{e2} \) given in table 1, while the total resistance is 3.3% smaller. Further refinement with grid 9 does not improve the prediction. The predicted result becomes much bigger than the expected value suddenly. Inspection of the numerical solution reveals that this sudden change in convergence behavior in grid 9 is caused by the thickness of viscous layer, which becomes too small. Thus, cells with poor quality in the transition region between the viscous layer and the outer layer deteriorate the accuracy of the numerical prediction in the viscous layer. The results in table 2 show the limitation of grid generation approach when using the unstructured grid generator.

Analysis of the predicted numerical solution reveals that the poor convergence behavior shown in table 2 is caused by an insufficient grid density near the trailing edge. In fact, a small recirculation zone is formed in this region. Higher grid resolution is required in this region in order to capture the flow separation correctly. We regenerate a new grid set by adding one more refinement level both around the leading edge and the trailing edge. On the coarsest grid 1, the mesh size is L/394 in those regions.

Table 3: Wall modeled prediction with mesh generation

<table>
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<tr>
<th>Inde</th>
<th>N1</th>
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<td>0.5064</td>
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</tr>
</tbody>
</table>

Table 3: Wall modeled prediction with mesh generation

Table 3 shows the predicted resistance as well as the extrapolated solution. The first two grids are too coarse. The extrapolation is obtained with the 5 finest solutions marked by a * in the table. For the pressure resistance, the observed order of convergence is 2.50. The extrapolated solution \( U_{e2} = 0.4173 \) differs from the corresponding result obtained with adaptive grid refinement shown in table 1 by less than 0.1%. This good agreement suggests that the prediction with the grid generation approach is now successful. However, on the finest grid 7, the predicted pressure resistance is still 2.7% smaller than the expected value, while with adaptive grid refinement, the prediction obtained with grid 4 with similar number of grid cells is only 0.5% smaller than \( U_{e2} \). For this computation, monotonic convergence behavior is also observed for the friction resistance with an observed order of convergence of 2.37. However, the predicted solutions do not change monotonously. Hence, this extrapolated result cannot be considered as reliable. Due to the poor convergence behavior in friction resistance, the observed order of convergence for the total resistance \( p=0.14 \) is too small. In this case, only the extrapolated solution with assumed second order accuracy \( U_{e2} \) can be used for uncertainty estimation. This extrapolated value differs from the value shown in table 1 by 0.3%. This good agreement confirms once again the good convergence behavior obtained with this grid set. On the finest grid 7, the predicted total resistance is 0.8% smaller than the extrapolated solution \( U_{e2} \).

4.2 WALL-RESOLVED SIMULATIONS

The adaptive grid refinement approach is also applied for wall-resolved computation. Results are shown in table 4. Monotonic convergence behavior is observed for all resistance components with an observed order of convergence higher than 3. This high value of the observed order of convergence may be due to the fact that in our computations, grids are not refined in the wall normal direction. Hence, grid similarity is not ensured. The predicted pressure resistance and total resistance are 0.53% and 0.26% higher than the extrapolated solution \( U_{e2} \) respectively, similar to what we obtained in the computation with wall function shown in table 1.

Table 4: Wall resolved prediction with adaptive refinement

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</table>

Table 4: Wall resolved prediction with adaptive refinement
Table 5: Wall resolved prediction with mesh generation

Table 5 shows the results obtained with wall-resolved simulations using grid set generated by the mesh generator Hexpress. Poor convergence behavior similar to the result shown in table 2 is observed. The predicted pressure and total resistance are smaller than the expected value. They increase as the grid is refined. But on the finest grid 7, the predicted pressure resistance and total resistance become larger than the expected value $U_{e2}$ shown in table 4. Again, this poor convergence behavior is due to the fact that grid similarity is not ensured in the near wall region with mesh generation approach. Moreover, poor quality cells next to the viscous layer deteriorate the accuracy of the numerical prediction in the viscous layer. It is very difficult to obtain a reliable uncertainty estimation for this simulation. Extrapolated results shown in table 5 are obtained with a selected grid triplets marked with a * in the table. The extrapolated $U_{e2}$ values differ from the results obtained with adaptive grid refinement given in table 4 by about 1% for all resistance components. However this extrapolation can not be considered as reliable.

5 CONCLUSIONS

Grid convergence studies for unstructured grids are investigated with different approaches using a simple 2D geometry. The newly proposed adaptive grid refinement approach is found to be capable of providing a solution with good convergence behavior both for wall function and for wall-resolved simulations. With the grid generation approach, since an unstructured grid generator is unable to generate a grid set ensuring grid similarity everywhere, user expertise is required when generating the grids. If the grid is well adapted to the flow, a good convergence behavior can be obtained. On the other hand, a poor convergence behavior is always an indication that the numerical prediction is not reliable. Such poor convergence behavior may be due to a too coarse grid resolution, or a poor grid quality locally. Generating an even finer grid cannot always improve the prediction. Compared with the usual approach of generating grid sets with a grid generator, the adaptive grid refinement approach is found to be more a reliable way of generating a similar grid set employed in a verification and validation exercise.

6 ACKNOWLEDGEMENTS

This work was granted access to the HPC resources under the allocation 2015-2a1308 made by GENCI (Grand Equipement National de Calcul Intensif).

7 REFERENCES


