Using Hemotopy Perturbation Method (HPM) to analysis 2D axial symmetric stagnation slip flow and heat transfer on a moving plate

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Abstract:

Analysis of the fluid flow and especially that of symmetric two-dimensional and incompressible stagnation flow with heat transfer has always been considered by scientists and researchers for a long time. During these analyses, nonlinear equations always create difficulties to solve the related problems and new methods of fluid flow analysis have proposed in the literature. In this paper, we propose a method to solve the axial symmetry stagnation flow equations including the equation of heat transfer, mass transfer and momentum equation. We use Hemotopy Perturbation Method to solve these equations. We plotted \( f \) as flow function, \( \theta \) as temperature and \( g \) as mass flux. In HPM we chose \( h=-1 \) for all functions and also \( H=1 \) for each of them. We compare our result by numerical results and found that the result have very well consistent. We use 50\(^{th}\) order for each function that are obtained from HPM. For 50\(^{th}\) order function our error is less than \( 10^{-6} \) that is showed in our plot and tables. Our good initial guesses help us to obtain functions in not high order functions.

Keywords: stagnation flow, axial symmetric flow, nonlinear equations, Hemotopy analysis method, HAM

1. Introduction and Related Work

Recently, finding analytical approximating solutions of nonlinear equations has widespread applications in numerical mathematics and applied mathematics and there has appeared an ever increasing interest of scientists and engineers in analytical techniques for studying nonlinear problems. Homotopy Analysis Method has been proposed by Liao and a systematic and clear exposition on this method is given in [1-2]. The study of viscous flow near a stagnation point was first proposed in 1911. Hiemenz [3] and Homann [4], gave the solutions for two-dimensional and the axial symmetric flows for a Newtonian fluid, respectively. Howarth [5] and Pavey [6] proposed the solution for a general three-dimensional flow. Stuart [7] investigated the two-dimensional oblique stagnation flow. Tamada [8] proposed an exact solution for the Navier–Stokes equations for steady two-dimensional oblique stagnation flow.

Here numerical discussion of the flow field is highlighted and comparison with an existing theory is presented. Dorrepaal [9] analyzed the two-dimensional but nonorthogonal stagnation-point flow and obtained the similarity solution. For Newtonian fluids the governing problems in
two dimensions for stagnation flow have a particularly simple solution, which has been given by Rott [10] but for the axisymmetric flow; Wang [11] solved the governing boundary value problem numerically. Libby [12] studied the three-dimensional stagnation flows on a moving plate. Weidman and Mahalingam [13] considered the axisymmetric stagnation-point flow impinging on an oscillating flat plate with a uniform suction. The solution to the arising problem is developed by two dimensionless groups, namely, the suction parameter and the frequency parameter. Numerical integrations by Runge-Kutta routine yield an exact solution to the Navier-Stokes equation. Recently Baris and Dokuz [14] presented an interesting study for three-dimensional stagnation-point flow of a second grade fluid toward a moving plate. They solved the governing problems numerically by employing the MATLAB solver singular boundary value problem [SBVP]. In all the above mentioned studies, no-slip conditions have been taken into account. However, there are situations wherein this condition does not hold. Mention may be made to the rarefied gases [15] coated surfaces such as Teflon, and resist adhesion. Navier [16] proposed a partial slip condition. In rarefied gases, there is a regime where Navier-Stokes equation holds in the presence of slip condition [15]. On the other hand the solid surface may be rough or porous such that equivalent slip is present [17]. In Ref. [17] the author studied the impinging stagnation flows toward a plate with Navier's slip condition by shooting method. In another paper Wang [18] analyzed the Stokes shear flow over a surface with evenly spaced, finite depth rectangular grooves by employing eigenfunction expansion method and matching. In continuation, Wang [19] discussed the influence of stagnation slip flow on the heat transfer from a moving plate. Numerical and asymptotic solutions of the governing equations are given in Ref[19]. The present paper investigates the homotopy analysis method [HAM] solution for the problem considered by Wang [19]. Different from perturbative and nonperturbative techniques, the HAM itself provides us with a convenient way to control and adjust the convergence region and the rate of approximation series when necessary. Liao [18] observed that Adomian decomposition method, -expansion method, and artificial small parameter method are all limited cases of HAM. HAM is a newly developed, powerful analytic technique, which has already been used by several investigators [21–39] for various interesting problems. Expressions for velocity and temperature fields are developed. The influence of partial slip on the flow and heat transfer characteristics is analyzed through graphs. Many detailed analysis is given in this area, including the analytical solutions can be pointed to the analysis of T. Javad Z. Abbas [40]. They have a good analysis for two-dimensional stagnation flow with axial symmetry with heat transfer and mass transfer using Homotopy analysis. In this paper a new solution method is introduced. for every linear equation has become in the reference [40] , boundary conditions are optimized.

Motivation and Contribution: The motivation behind this work is to find an efficient and fast method to solve the nonlinear equations in homotopy analysis and ..... Our contribution in this paper is: first, we propose boundary conditions for each function (F0, F1, F2, ..) of the initial guess. Then, we solve the nonlinear equations of heat transfer, mass transfer and momentum equation based on these boundary conditions. The proposed method is implanted in Maple and compared with other methods which are based on Zero condition for the linear equation. The results show that our approach has
13% better runtime in average for these three equations.

2. Problem Description

Problem is about the two-dimensional and incompressible stagnation flow, which is on the plate flow in x-z coordinates. Plate is in the position z = 0 with speed U and V in the x and z directions, respectively. According to reference [38] equations of momentum, heat transfer and mass transfer are as follows:

\[ f''' + ff' - f'^2 + 1 = 0 \]  
\[ \theta''' + Prf\theta' = 0 \]  
\[ g''' + fg' - f'g = 0 \]  
\[ K''' + fK' = 0 \]

Boundary conditions are:

\[ f'(0) = 1 + \lambda f''(0) \]  
\[ g(0) = 1 + \lambda g'(0) \]  
\[ K(0) = 1 + \lambda K'(0) \]  
\[ \theta(0) = 1 + \beta \theta'(0) \]  
\[ f'(\infty) = 1, g(\infty) = 0, \]  
\[ f(0) = 0, \theta(\infty) = 0 \]

Here \( \lambda = N \sqrt{\frac{\rho}{\nu}} \) is the nondimensional slip factor, \( \beta = \frac{S}{\sqrt{v}} \) is the nondimensional thermal slip parameter, \( \nu \) is the kinematic viscosity, \( a \) is the strength of the stagnation flow, \( N, v, S \) the slip constant, kinematic viscosity, the constant of proportionality for thermal slip respectively and \( Pr \) is the Prandtl number. In a two-dimensional flow, for an impermeable plate we take \( f(0)=0 \).

3. Optimization equations for the axial symmetric stagnation flow

During the axial symmetry due to symmetry, the equations 1 to 4 will be little changed and the boundary conditions can also change the overall equation is as follows. [38]

\[ f''' + 2ff' - f'^2 + 1 = 0 \]  
\[ g''' + 2fg' - f'g = 0 \]  
\[ \theta''' + 2Prf\theta' = 0 \]

Boundary conditions are:

\[ f'(0) = 1 + \lambda f''(0) \]  
\[ g(0) = 1 + \lambda g'(0) \]  
\[ \theta(0) = 1 + \beta \theta'(0) \]  
\[ f'(\infty) = 1, g(\infty) = 0, \]  
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4. Solution

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homotopy based method

HAM has been established based on Homotopy. Homotopy is a principle in topology and differential equations. If we simply want to explain Homotopy analysis, can be said by the Homotopy analysis can be evaluated by a series of initial guess and by a series of auxiliary parameters obtained a series of solutions that converge to The exact solution. This method uses features such as freedom to choose the initial function and linear operator uses. By the same freedoms and basic choices, a complex nonlinear problem is solved and the problem becomes less linear and simple. So if the differential equation is defined as follows:
N\left( V(r,t) \right) = 0

N is a nonlinear operator. Considering the linear operator L, non-zero auxiliary functions h, H and the initial guess \( V_0(r,t) \), non-linear equations (8) can be written into the following equation:

\[
(1 - q)L[\varphi(r,t; q) - V_0(r,t)] = qhH(r,t)N[\varphi(r,t; q)]
\]

In equation (9) 0 ≤ q ≤ 1. The following equations are established. In Hemotopy perturbation method (HPM) we choose \( h=-1 \) and \( H=1 \).

\[
\varphi(r,t; 0) = V_0(r,t)
\]

\[
\varphi(r,t; 1) = V(r,t)
\]

In this way, when q changes from 0 to 1, the initial guess function \( V_0(r,t) \) will converge to the function \( V(r,t) \) (with regard to convergence conditions).

Finally, considering the Taylor expansion of \( \varphi(r,t; q) \) by changing q around zero, and several algebraic operations, the following equation is written for m-order of \( V(r,t) \).

\[
V(r,t) \approx \sum_{i=0}^{m} V_i(r,t)
\]

Or in other words:

\[
V(r,t) \approx V_0(r,t) + V_1(r,t) + \cdots + V_m(r,t)
\]

Note that in this way, auxiliary functions, the initial guess and the linear operator can be selected, so the freedom to choose guesses in this way is high. But the initial guess and auxiliary functions are chosen in such a way that may fit with boundary conditions and convergence issue.

Select linear operator, the initial guess, boundary conditions and non-linear operators For equation (5) the highest derivative degree is 3, the linear operator is selected as follows:

\[
I_f = \gamma_1 \frac{\partial^3 f}{\partial \eta^3} + \gamma_2 \frac{\partial^2 f}{\partial \eta^2} + \gamma_3 \frac{\partial f}{\partial \eta} + \gamma_4 f
\]

If \( \gamma_2 \) and \( \gamma_4 \) are opposed to zero, Genesis sentences with \( \eta \wedge n \) (n>1) in the function f that According to boundary conditions, the equation is causing divergence. So choosing zero \( \gamma_2 \) and \( \gamma_4 \), Option 1 and -1 for \( \gamma_1 \) and \( \gamma_3 \) respectively. We choose the linear operator \( L_f \) like what reference [38] is given.

\[
I_f = \frac{\partial^3 f}{\partial \eta^3} - \frac{\partial f}{\partial \eta}
\]

With The same arguments as for other linear operators we choose:

\[
I_g = \frac{\partial^2 g}{\partial \eta^2} - g
\]

\[
I_\theta = \frac{\partial^2 \theta}{\partial \eta^2} - \theta
\]

Non-linear operators are thus:

\[
N_f = \frac{\partial^3 f}{\partial \eta^3} + 2f \frac{\partial^2 f}{\partial \eta^2} - \left( \frac{\partial f}{\partial \eta} \right)^2 + 1
\]

\[
N_g = \frac{\partial^2 g}{\partial \eta^2} + 2f \frac{\partial g}{\partial \eta} - g \frac{\partial f}{\partial \eta}
\]

\[
N_\theta = \frac{\partial^2 \theta}{\partial \eta^2} + 2Pr \frac{\partial \theta}{\partial \eta}
\]

A_0 = 1, \( \gamma_0 \), \( \gamma_1 \) and \( \gamma_2 \) for 0 ≤ m ≤ 1 are determined from the following equation:

\[
L[\chi_m(r,t) - \chi_m \cdot V_0(r,t)] = hH(r,t)\sum_{i=0}^{m} \chi_m [V_{m-1}(r,t)]
\]

\[
\chi_m \begin{cases} 0 & m = 1 \\ 1 & m > 1 \\ \end{cases}
\]

It should be mentioned that the boundary conditions given for the main functions f, g and \( \theta \)
Therefore, for each of the index functions, boundary conditions must be optimized. Boundary conditions are optimal for each of the following equations:

For function $f$ boundary conditions are thus:

\[
f'(0) = 1 + \lambda f''(0) \\
f'(\infty) = 1, f(0) = 0
\]

Expression $1 + \lambda f''(0)$ is a constant, a value for it to be considered and:

\[
f(\eta) = f_0(\eta) + f_1(\eta) + f_2(\eta) + \cdots + f_m(\eta)
\]

Two conditions to be considered in the equation:

\[
f_m(0) = 0 \\
f'_m(0) = 0
\]

$A_1$ is a constant that is determined at any stage. $f'(\infty) = 1$ at the end of each phase are checked.

With similar reasoning for functions $g$ and $\theta$:

\[
g_m(\infty) = 0 \\
g_m(0) = 0 \\
\theta_m(\infty) = 0 \\
\theta_m(0) = 0
\]

Values $B_3$ and $C_1$ are determined at the end of each stage.

Using Maple software following values were obtained in the first stage:

\[
f_1(\eta) = \left(\frac{1}{6+(1+\lambda)^3}\right)\left[3\lambda^2 h \exp(-\eta) + 6\lambda h\exp(-\eta) + 3\exp(-\eta)\right] + [18\lambda h \exp(-\eta) + 6\exp(-\eta) + 12\lambda^2 h\exp(-\eta) + 7\lambda^2 + 18\lambda^2 h\exp(-\eta) - 7\lambda^2 - 18h\lambda^2 + h\lambda\exp(-2\eta) + 6\lambda^2\exp(-\eta) - 12\lambda\exp(-\eta) - 21h + 12\lambda + 22h\exp(-\eta) - 6\exp(-\eta) + 8\exp(-\eta) + 6]
\]

According to the above equation, function $f$ is based on $\eta^k \exp(-\eta)$ Which is consistent with the series introduced in reference [40].

For functions $g$ and $\theta$, the following values are obtained in the first stage.

\[
g_1(\eta) = \left(\frac{1}{6+(1+\lambda)^3}\right)\left[3\lambda^2 h \exp(-\eta) + 6\lambda h\exp(-\eta) + 3\exp(-\eta)\right] \eta^2 + [-3\exp(-\eta) + 3h \exp(-\eta)] \eta + [3h\lambda^2 \exp(-\eta) - 2h\exp(-2\eta) + 2\exp(-2\eta) + 6\exp(-\eta) + 6\lambda \exp(-\eta) + 6\exp(-\eta) + 2h\exp(-\eta)]
\]

and

\[
\theta_1 = \left(\frac{1}{6+(1+\lambda)^3}\right)\left[(3hPr\lambda^2 + 6hPr\lambda + 3hPr)\eta^2 \exp(-\eta) + (-3h\lambda + 3hPr\lambda^2 - 6h\lambda - 3hPr - 3h\eta \exp(-\eta) + (-3h\lambda^2 + 3hPr\lambda^2 + 4hPr + 5hPr\lambda - 3h\lambda - 4h\lambda Pr + 6\lambda - 4hPr + 6)\exp(-\eta)]
\]

Analysis of results and diagrams:

Figure 1: 50th order of $f$ function is showed by line and numerical results by blue points in term of $\eta$. $Pr=1, \lambda = \beta = 0.1$
We change Prandtl number to obtain effect of this dimensionless number on each flow parameters. As show as in Figure 4 changing prandtl effected only on temperature.

Figure 2: 50th order of $\theta$ function is showed by line and numerical results by blue points in term of $\eta$. $\text{Pr}=1$, $\lambda = \beta = 0.1$

Figure 3: 50th order of $\theta$ function is showed by line and numerical results by blue points in term of $\eta$. $\text{Pr}=1$, $\lambda = \beta = 0.1$

Figure 4: effect of prandtl variation of temperature. blue line $\text{Pr}=6$, green line $\text{Pr}=3$ and red line $\text{Pr}= 1$. $\lambda = \beta = 0.1$

Figure 5: effect of $\lambda$ and $\beta$ flow parameter. $\lambda = \beta = 0.1$ red line and $\lambda = \beta = 0.01$ in black. $\text{Pr}=1$
\( \lambda \) and \( \beta \) are two parameters that effect on each functions and their effect is showed in Figure 5.

For \( \lambda = \beta = 0.1 \) and \( \lambda = \beta = 0.01 \) (black line), we plot the effect of these parameters.

![Figure 6: effect of \( \lambda \) and \( \beta \) on temperature. \( \lambda = \beta = 0.1 \) red line and \( \lambda = \beta = 0.01 \) in black. Pr=1](image)

![Figure 7: effect of \( \lambda \) and \( \beta \) on mass flux parameter. \( \lambda = \beta = 0.1 \) red line and \( \lambda = \beta = 0.01 \) in black. Pr=1](image)

Conclusion:

Our results is showed in figures 1-3. These figures show the good consistent between our 50-order functions and numerical results. We obtain errors less than \( 10^{-5} \) for each 50-order functions. We use an excellent Maple code to obtain these results so we employ a high speed computer to obtain 50-order functions. HPM is good for solve nonlinear equations but in some cases we need to employ very high speed computer to solve a system of equations that are couple with each other. Our good initial guesses help us to converge in not high order functions. Increasing prandtl number only effect on temperature. Its effect is on increasing speed of convergence to the ambeint temperature. Variation of prandtl is no effect on mass flux and velociy feild. variation of \( \lambda \) and \( \beta \) effect on \( \theta, \theta_0, \theta_1 \) (velocity, temperature and mass flux non-dimensional functions).

References:


[27] Liao, S. J., 2005, “Comparison Between the Homotopy Analysis Method and Homotopy


